Clarke's and Park's Transformations

BPRA047, BPRA048

Content

Introduction

- The performance of three-phase AC machines are described by their voltage equations and inductances.
  - It is well known that some machine inductances are functions of rotor speed.

$$(V_S) = \begin{bmatrix} v_{s1} \\ v_{s2} \\ v_{s3} \end{bmatrix}$$

- The coefficients of the differential equations, which describe the behavior of these machines, are time varying except when the rotor is stalled.

Introduction

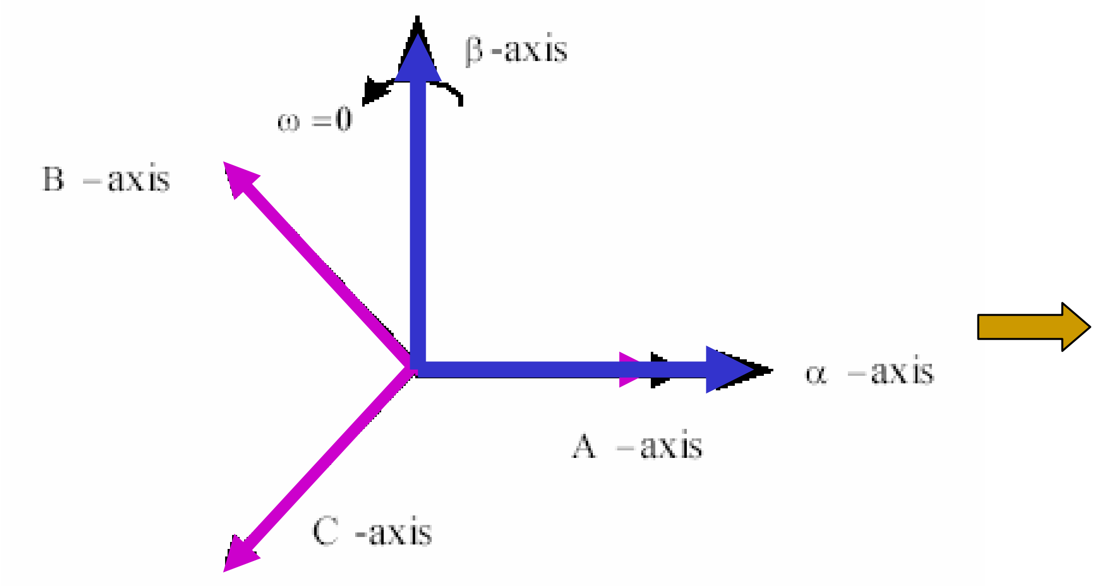
- A change of variables is often used to reduce the complexity of these differential equations.

- In this chapter, the well-known Clarke and Park transformations are introduced, modeled, and implemented on the LF2407 DSP.

- Using these transformations, many properties of electric machines can be studied without complexities in the voltage equations.

Clarke's Transformation

- The transformation of stationary circuits to a stationary reference frame was developed by E. Clarke.
- The stationary two-phase variables of Clarke's transformation are denoted as α and β, α-axis and β-axis are orthogonal.



$$v_\alpha = v_a$$

$$v_\beta = \frac{2v_b + v_a}{\sqrt{3}}$$

Clarke's Transformation

- In order for the transformation to be invertible, a third variable, known as the zero-sequence component, is added.
- The resulting transformation is

$$\left[f_{\alpha\beta 0}\right] = T_{\alpha\beta 0}\left[f_{abc}\right]$$

where *f* represents voltage, current, flux linkages, or electric charge; the transformation matrix T:

$$T_{\alpha\beta 0} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Inverse Clarke's Transformation

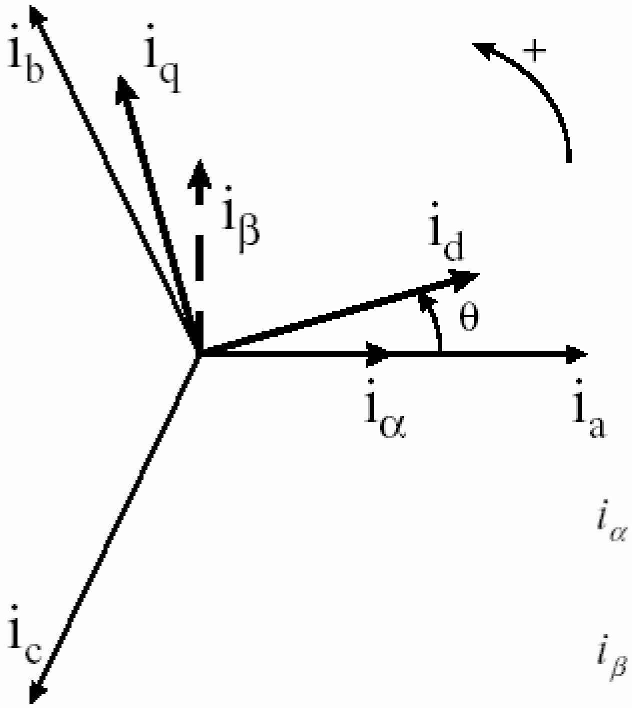
- The inverse transformation is given by

$$[f_{abc}] = T_{\alpha\beta 0}{}^{-1}[f_{\alpha\beta 0}]$$

where the inverse transformation matrix T^{-1} is presented by

$$T_{\alpha\beta 0}{}^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix}$$

Clarke's Transformation



$$i_\alpha = \frac{2}{3}\cdot i_a - \frac{1}{3}(i_b - i_c)$$

$$i_\beta = \frac{2}{\sqrt{3}}(i_b - i_c)$$

$$i_o = \frac{2}{3}(i_a + i_b + i_c)$$

$$i_\alpha = i_a$$

$$i_\beta = \frac{1}{\sqrt{3}}\cdot i_a + \frac{2}{\sqrt{3}} i_b$$

$$i_a + i_b + i_c = 0$$

$$i_a = i_\alpha$$

$$i_b = -\frac{1}{2}\cdot i_\alpha + \frac{\sqrt{3}}{2}\cdot i_\beta$$

$$i_c = -\frac{1}{2}\cdot i_\alpha - \frac{\sqrt{3}}{2}\cdot i_\beta$$

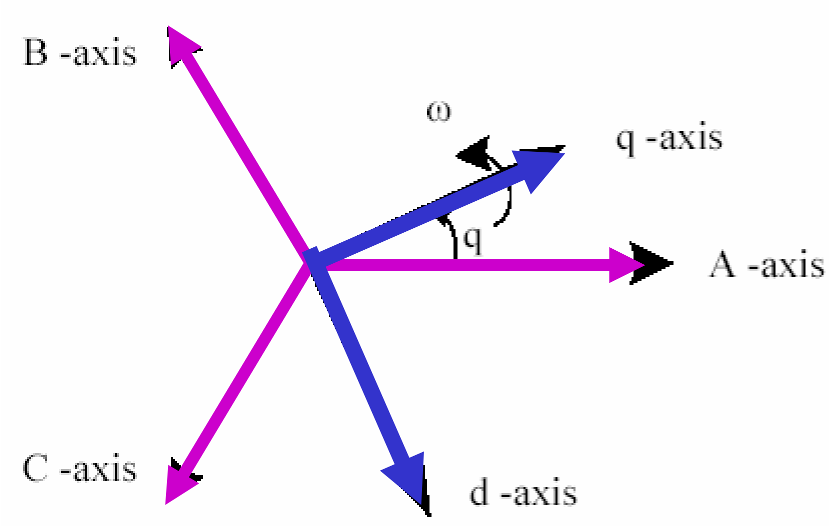
Park's Transformation

- In the late 1920s, R.H. Park introduced a new approach to electric machine analysis.
- He formulated a change of variables associated with fictitious windings rotating with the rotor.
- He referred the stator and rotor variables to a reference frame fixed on the rotor.
- From the rotor point of view, all the variables can be observed as constant values.

- Park's transformation, a revolution in machine analysis, has the unique property of eliminating all time varying inductances from the voltage equations of three-phase ac machines due to the rotor spinning.

Park's Transformation

- Park's transformation is a well-known three-phase to two-phase transformation in synchronous machine analysis.



$$\begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix} = \begin{bmatrix} \cos\theta_S & -\sin\theta_S & 1 \\ \cos(\theta_S - \frac{2\pi}{3}) & -\sin(\theta_S - \frac{2\pi}{3}) & 1 \\ \cos(\theta_S - \frac{4\pi}{3}) & -\sin(\theta_S - \frac{4\pi}{3}) & 1 \end{bmatrix} \begin{bmatrix} v_{sd} \\ v_{Sq} \\ v_{So} \end{bmatrix}$$

$$\begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix} = [P(\theta_S)] \cdot \begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{so} \end{bmatrix} \text{ and } \begin{bmatrix} v_{sd} \\ v_{Sq} \\ v_{So} \end{bmatrix} = [P(\theta_S)]^{-1} \begin{bmatrix} v_{s1} \\ v_{S2} \\ v_{S3} \end{bmatrix}$$

Park's Transformation

- The Park's transformation equation is of the form

$$\left[f_{qd0s}\right] = T_{qd0}(\theta)\left[f_{abcs}\right]$$

$$T_{qd0s}(\theta) = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

- θ is the angular displacement of Park's reference frame

Inverse Park's Transformation

- It can be shown that for the inverse transformation we can write

$$[f_{abcs}] = T_{qd0}(\theta)^{-1} \cdot [f_{qd0s}]$$

where the inverse of Park's transformation matrix is given by

$$T_{qd0}(\theta)^{-1} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix}$$

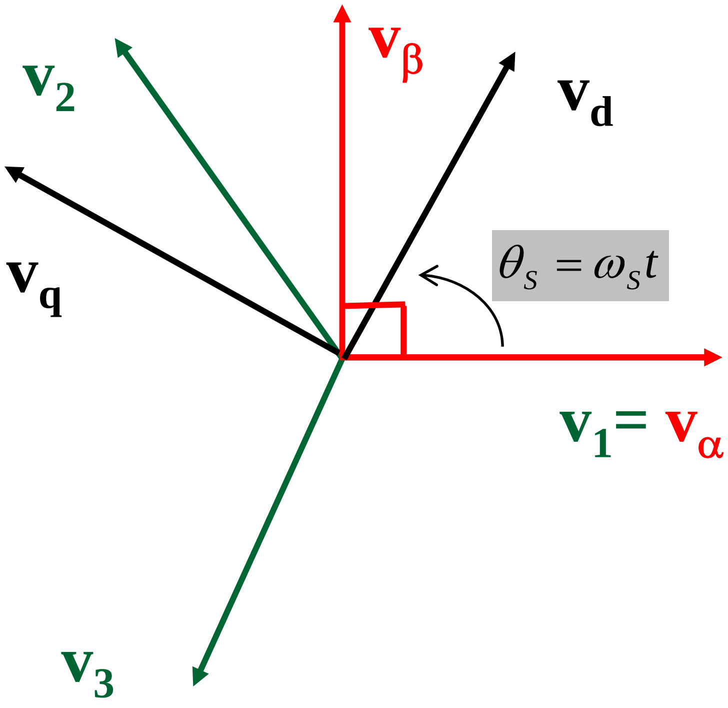
Park's Transformation

- The angular displacement θ must be continuous, but the angular velocity associated with the change of variables is unspecified.
- The frame of reference may rotate at any constant, varying angular velocity, or it may remain stationary.
- The angular velocity of the transformation can be chosen arbitrarily to best fit the system equation solution or to satisfy the system constraints.

- The change of variables may be applied to variables of any waveform and time sequence;
  - however, we will find that the transformation given above is particularly appropriate for an a-b-c sequence.

Park's Transformation

- Park's Transform is usually split into CLARKE transform and one rotation;
- CLARKE converts balanced three phase quantities into balanced two phase orthogonal quantities;



CLARKE

$$v_\alpha = v_1$$

$$v_\beta = \frac{2.v_2 + v_1}{\sqrt{3}}$$

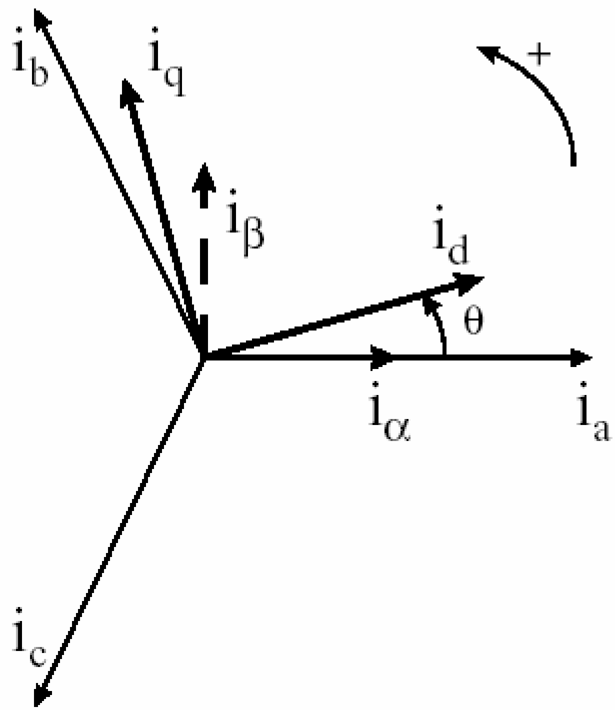
+

Rotation

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos(\theta_S) & \sin(\theta_S) \\ -\sin(\theta_S) & \cos(\theta_S) \end{bmatrix} \cdot \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}$$

PARK TRANSFORM

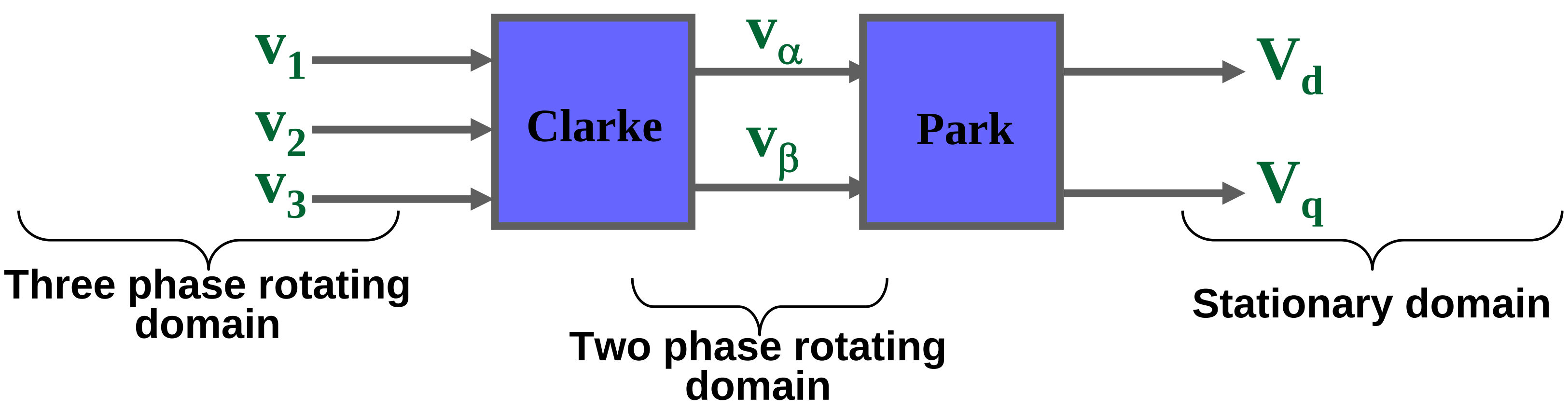
Park’s Transformation



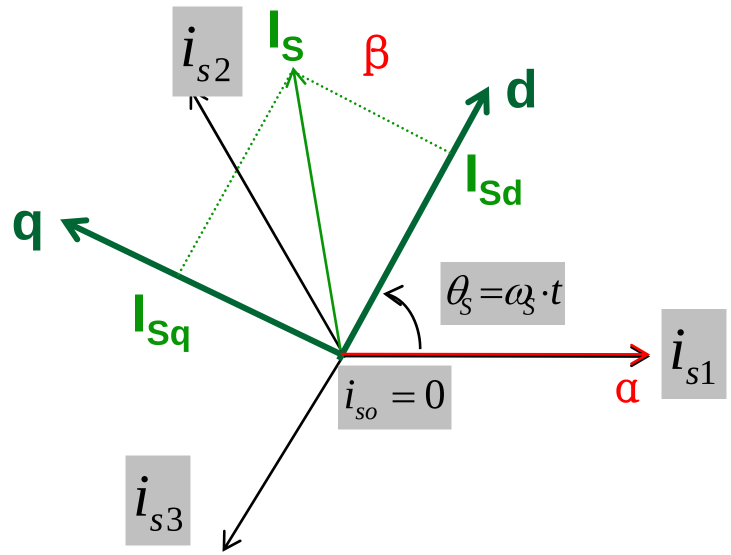
$$i_{sd} = i_\alpha \cdot \cos(\theta) + i_\beta \cdot \sin(\theta)$$
$$i_{sq} = -i_\alpha \cdot \sin(\theta) + i_\beta \cdot \cos(\theta)$$

$$i_\alpha = i_{sd} \cdot \cos(\theta) - i_{sq} \cdot \sin(\theta)$$
$$i_\beta = i_{sd} \cdot \sin(\theta) + i_q \cdot \cos(\theta)$$

Transform summary

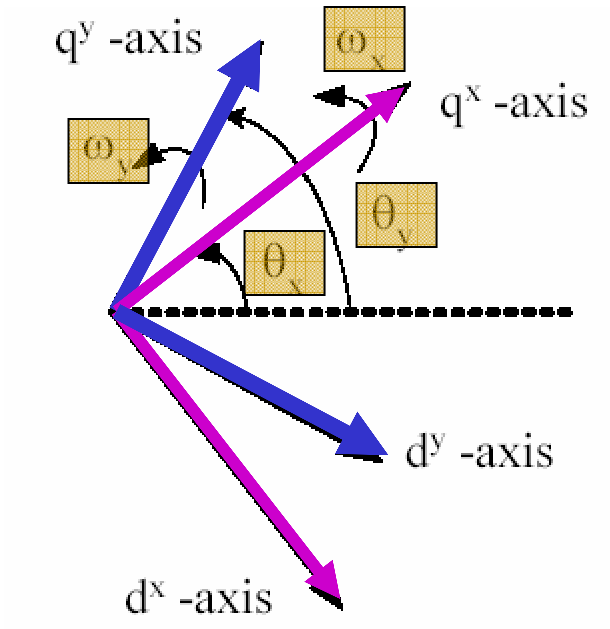


- Stator phase current example:
- I_s is moving at θ_s and its PARK coordinates are constant in (d,q) rotating frame.
- Can be applied on any three-phase balanced variables (flux…)



Transformations Between Reference Frames

- In order to reduce the complexity of some derivations, it is necessary to transform the variables from one reference frame (d^x, q^x) to another one (d^y, q^y) .



Transformations Between Reference Frames

- In this regard, we can rewrite the transformation equation as

$$\left[f_{qd0s}^{y}\right] = T_{qd0s}^{x \to y} \cdot \left[f_{qd0s}\right]$$

- But we have

$$\left[f_{qd0s}^{x}\right] = T_{qd0s}^{x} \cdot \left[f_{abcs}\right]$$

- we get

$$\left[f_{qd0s}^{y}\right] = T_{qd0s}^{x \to y} \cdot T_{qd0s}^{x} \cdot \left[f_{abcs}\right]$$

Transformations Between Reference Frames

- In another way, we can find out that

$$\left[f_{qd0s}^{y}\right]=T_{qd0s}^{y}\cdot\left[f_{abcs}\right]$$

- we obtain

$$T_{qd0s}^{x\to y}=T_{qd0s}^{y}\cdot T_{qd0s}^{x}{}^{-1}$$

- Then, the desired transformation can be expressed by the following matrix:

$$T_{qd0s}^{x\to y}=\begin{bmatrix}\cos(\theta_y-\theta_x) & -\sin(\theta_y-\theta_x) & 0\\ \sin(\theta_y-\theta_x) & \cos(\theta_y-\theta_x) & 0\\ 1 & 1 & 1\end{bmatrix}$$

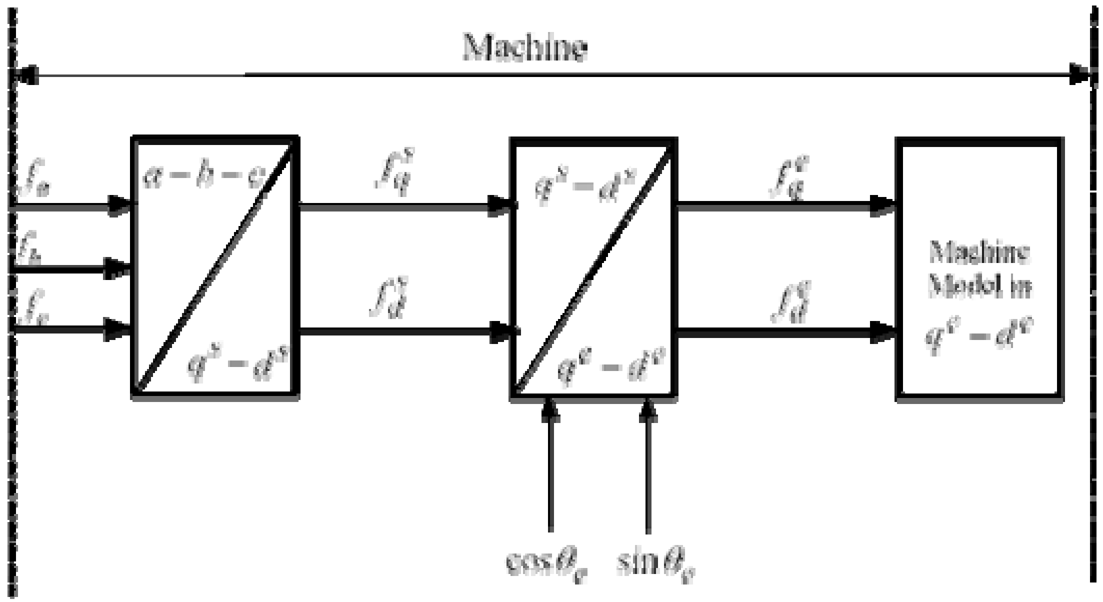
Field Oriented Control (FOC) Transformations

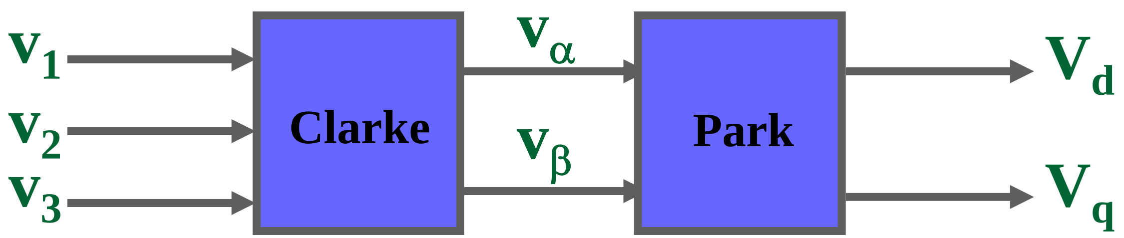
- In the case of FOC of electric machines, control methods are performed in a two-phase reference frame fixed to the rotor (q^r-d^r) or fixed to the excitation reference frame (q^e-d^e).

- We want to transform all the variables from the three-phase a-b-c system to the two-phase stationary reference frame and then retransform these variables from the stationary reference frame to a rotary reference frame with arbitrary angular velocity of ω.

Field Oriented Control (FOC) Transformations

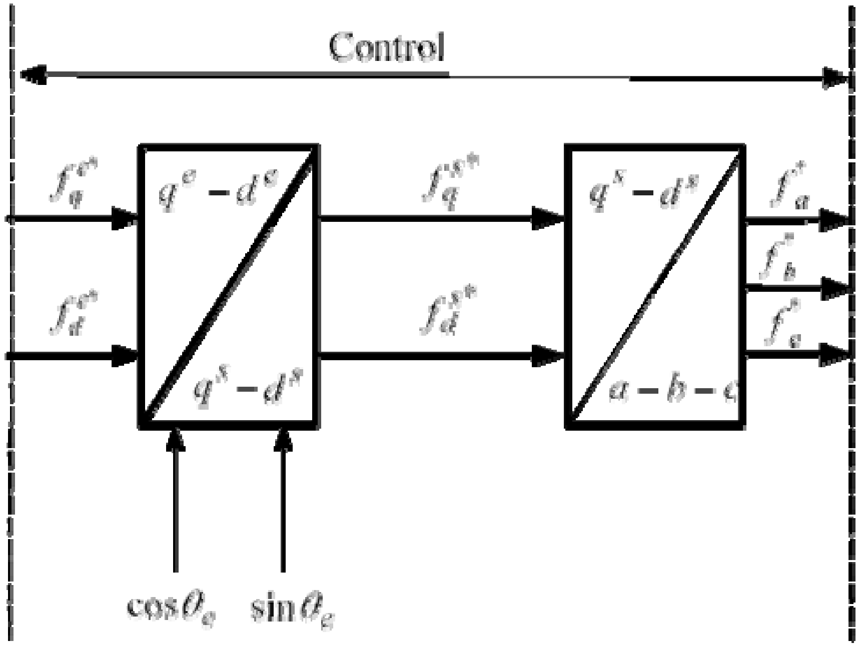
- These transformations are usually cascaded, the block diagram of this procedure is shown:





Field Oriented Control (FOC) Transformations

- In the vector control method, after applying field- oriented control it is necessary to transform variables to stationary a-b-c system.
- This can be achieved by taking the inverse transformation.



Implementing Clarke's Transformation

- Clarke's Transformation is to transfer the three-phase stationary parameters from a-b-c system to the two-phase stationary reference frame.
- It is assumed that the system is balanced

$$f_a + f_b + f_c = 0$$

we have

$$f_\alpha = \frac{2}{3} f_a - \frac{1}{3} f_b - \frac{1}{3} f_c$$

$$f_\beta = \frac{1}{\sqrt{3}} (f_b - f_c)$$

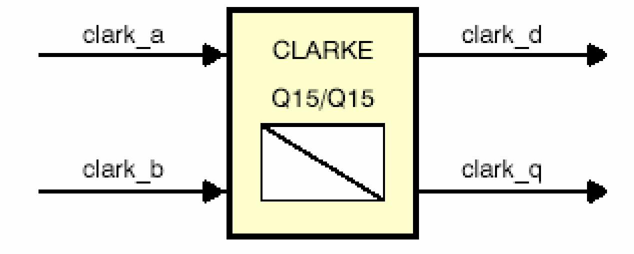
Then we get:

$$f_\alpha = f_a$$

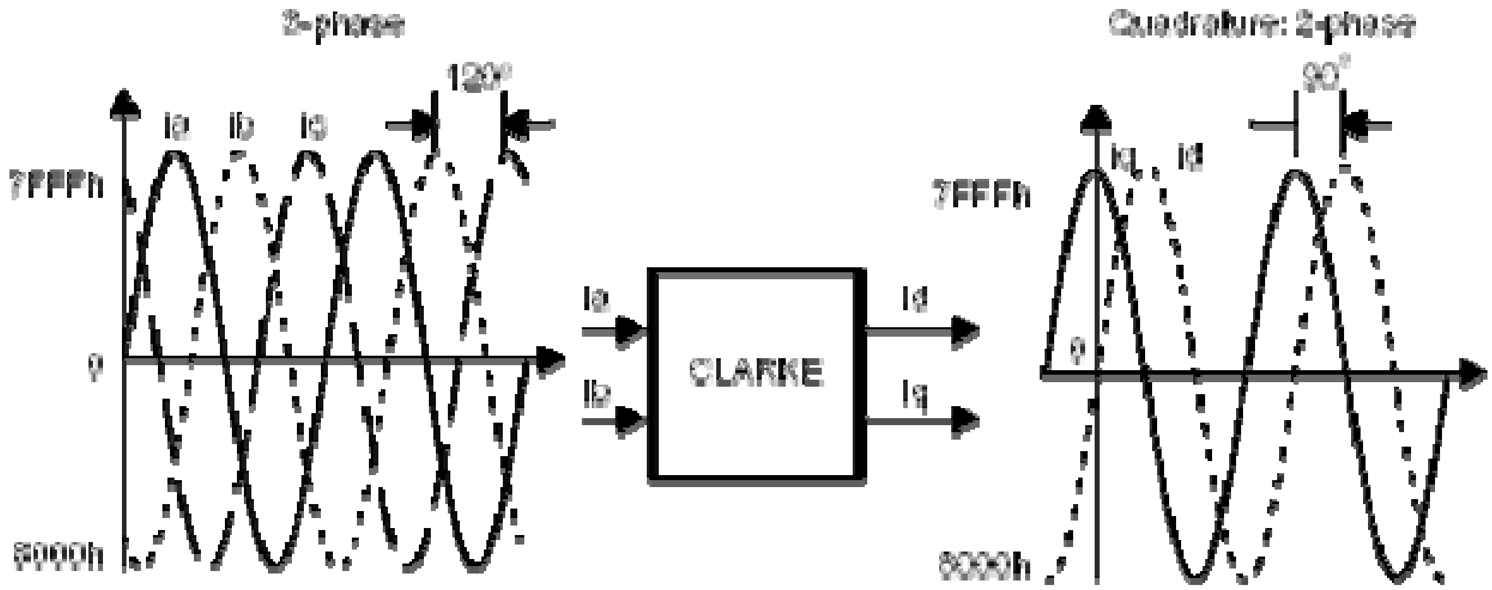
$$f_\beta = \frac{1}{\sqrt{3}} (f_a + 2 f_b)$$

Clarke's Transformation in DSP

- Clarke's Transformation Converts balanced three phase quantities into balanced two phase quadrature quantities.



Clarke's Transformation in DSP



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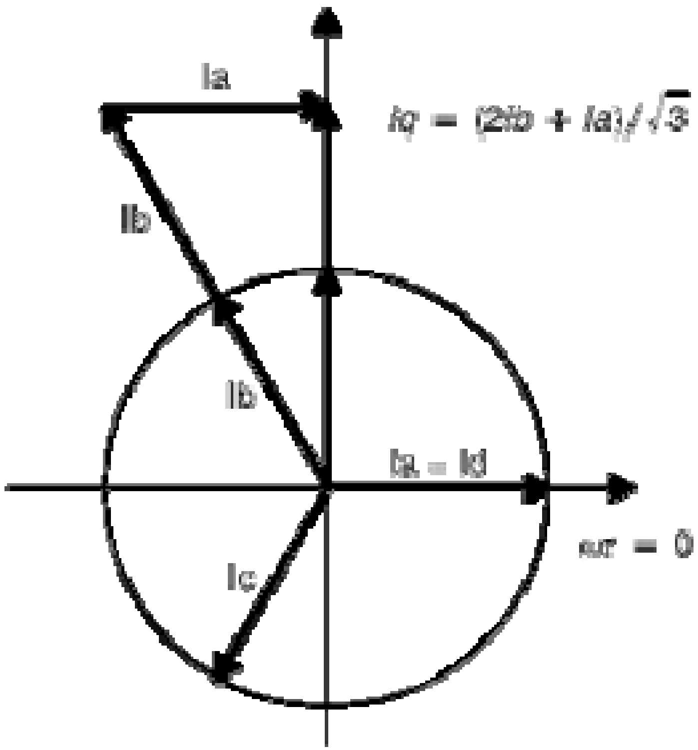
Clarke's Transformation in DSP

The instantaneous input and the output quantities are defined by the following equations:

$$ia = I \times \sin(\omega t)$$
$$ib = I \times \sin(\omega t + 2\pi/3)$$
$$ic = I \times \sin(\omega t - 2\pi/3)$$
$$\begin{cases} id = I \times \sin(\omega t) \\ iq = I \times \sin(\omega t + \pi/2) \end{cases}$$



Clarke's Transformation in DSP

- To enjoy better resolution of the variables in fixed point DSP, we transfer all variables to the Q15-based format.
- With this consideration, the maximum value of inputs and outputs can be ($2^{15}-1$) or 7FFFh in hexadecimal.
- In this base, the variables can vary in the range 8000h-7FFFh.

- Then 1/ √ 3 is represented by

```
LDP     #sqrt3inv            ;sqrt3inv=(1/sqrt(3))
                             ;=0.577350269
SPLK    #018918,sqrt3inv     ;1/sqrt(3) (Q15)
                             ;=0.577350269*2^15
```

Clarke's Transformation in DSP

- Clarke's transformation is implemented as follows:

```
SETC    SXM              ;Sign extension mode on
LDP     #clark_a         ;clark_alfa = clark_a
LACC    clark_a          ;ACC = clark_a
SACL    clark_alfa       ;clark_d = clark_a
                         ;clark_beta=(2*clark_b+clark_a)/
                         ;sqrt(3)
SFR                      ;ACC = clark_a/2
ADD     clark_b          ;ACC = clark_a/2 + clark_b
SACL    clk_temp         ;clk_temp = clark_a/2 + clark_b
LT      clk_temp         ;TREG = clark_a/2 + clark_b
MPY     sqrt3inv         ;PREG=(clark_a/2+clark_b)*
                         ;(1/sqrt(3))
PAC                      ;ACC=(clark_a/2+clark_b)*
                         ;(1/sqrt(3))
SFL                      ;ACC=(clark_a+clark_b*2)*
                         ;(1/sqrt(3))
SACH    clark_beta       ;clark_beta=(clark_a+clark_b*2
                         ;(1/sqrt(3))
SPM     0                ;SPM reset
RET
```

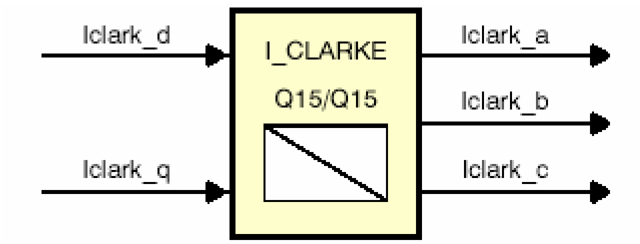
Implementing Inverse Clarke's Transformation

- The inverse Clarke functions Converts balanced two phase quadrature quantities into balanced three phase quantities.

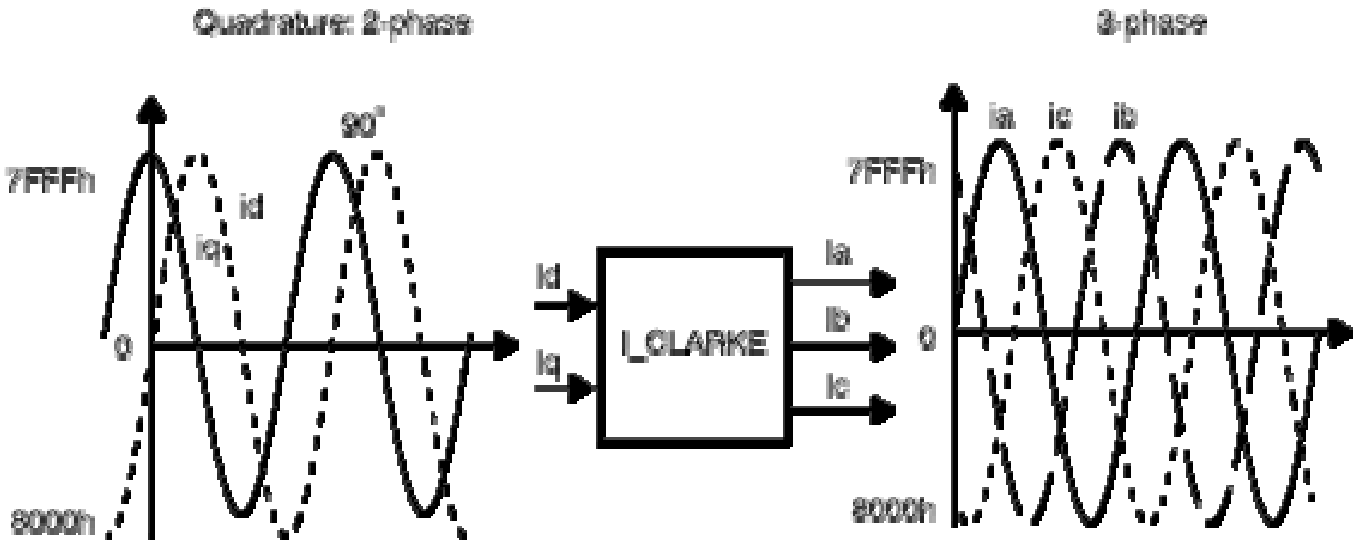
$$f_a = f_\alpha$$

$$f_b = \frac{-f_\alpha + \sqrt{3} * f_\beta}{2}$$

$$f_c = \frac{-f_\alpha - \sqrt{3} * f_\beta}{2}$$

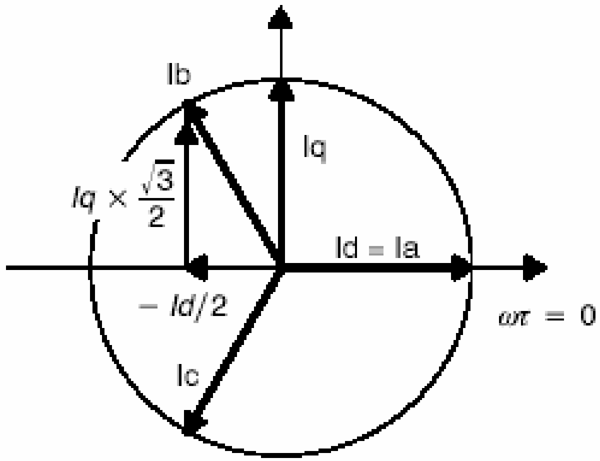


Implementing Inverse Clarke’s Transformation



$$\begin{cases} id = I \times \sin(\omega t) \\ iq = I \times \sin(\omega t + \pi/2) \end{cases}$$

$$\begin{cases} ia = I \times \sin(\omega t) \\ ib = I \times \sin(\omega t + 2\pi/3) \\ ic = I \times \sin(\omega t - 2\pi/3) \end{cases}$$



Implementing Inverse Clarke's Transformation

```
I_CLARKE_INIT:
        LDP     #half_sqrt3             ;Variables data page
        SPLK    #28377,half_sqrt3       ;Set constant sqrt(3)*0.5 in Q15
                                        ;format
RET
I_CLARKE:
        LDP     #f_clark_alpha          ;Variables data page
        SPM     1                       ;SPM set for Q15 multiplication
        SETC    SXM                     ;Sign extension mode on
        LACC    f_clark_alpha           ;ACC = f_alpha
        SACL    f_clark_a               ;f_a = f_alpha
        LT      f_clark_beta            ;TREG = f_clark_beta
        MPY     half_sqrt3              ;PREG=f_clark_beta * half_sqrt3
        PAC                             ;ACC= f_clark_beta * half_sqrt3
SUB     f_clark_alpha,15                ;ACC=f_beta*half_sqrt3-f_alpha/2
        SACH    f_clark_b
        PAC                             ;ACC high = f_beta*half_sqrt3
        NEG                             ;ACC high = - f_beta*half_sqrt3
        SUB     f_clark_alpha,15        ;ACC high=-f_beta*half_sqrt3-
                                        ;f_alpha/2
        SACH    f_clark_c               ;f_c = - f_beta * half_sqrt3 -
                                        ;f_alpha/2
        SPM     0                       ;SPM reset
        CLRC    SXM                     ;Sign extension mode off
RET
```

Calculation of Sine/Cosine

- To implement the Park and the inverse Park transforms, the sine and cosine functions need to be implemented.
- This method realizes the sine/cosine functions with a look-up table of 256 values for 360° of sine and cosine functions.
- The method includes linear interpolation with a fixed step table to provide a minimum harmonic distortion.
- This table is loaded in program memory.
- The sine value is presented in Q15 format with the range of $-1<value<1$.

Calculation of Sine/Cosine

- The first few rows of the look-up sine table are presented as follows:

```
;SINVALUE          ;      Index    Angle    Sin(Angle)
----------------          --------  --------  ------------
SINTAB_360
.word      0       ;      0        0        0.0000
.word      804     ;      1        1.41     0.0245
.word      1608    ;      2        2.81     0.0491
.word      2410    ;      3        4.22     0.0736
.word      3212    ;      4        5.63     0.0980
```

Calculation of Sine/Cosine

- The following assembly code is written to read values of sine from the sine Table in Q15 format:

```
LACC    theta_p, 9       ;Input angle in Q15 format and
                         ;left shifted by 15
SACH    t_ptr            ;Save high ACC to t_ptr (table
                         ;pointer)
LACC    #SINTAB_360
ADD     t_ptr
TBLR    sin_theta        ;sin_theta = Sin(theta_p) in Q15
```

Calculation of Sine/Cosine

- Note that 0 < theta_p < 7FFFh (i.e., equivalent to 0 < theta_p < 360 deg).
- The TBLR instruction transfers a word from a location in program memory to a data-memory location specified by the instruction.
- The program-memory address is defined by the low-order 16 bits of the accumulator.
- For this operation, a read from program memory is performed, followed by a write to data memory.

Calculation of Sine/Cosine

- To calculate the cosine values from the sine Table in Q15 format, we write the following code:

```
LACC    theta_p
ADD     #8192           ;add 90 deg, cos(A)=sin(A+900)
AND     #07FFFh         ;Force positive wrap-around
SACL    GPR0_park       ;here 90 deg = 7FFFh/4
LACC    GPR0_park,9
SACH    t_ptr
LACC    #SINTAB_360
```

Implementation of Park's Transformation

- With field-oriented control of motors, it is necessary to transform variables,
  - from a-b-c system to two-phase stationary reference frame, *qs-ds*,
  - and from two-phase stationary reference frame *qs-ds* to arbitrary rotating reference frame with angular velocity of ω (*q-d* reference frame).

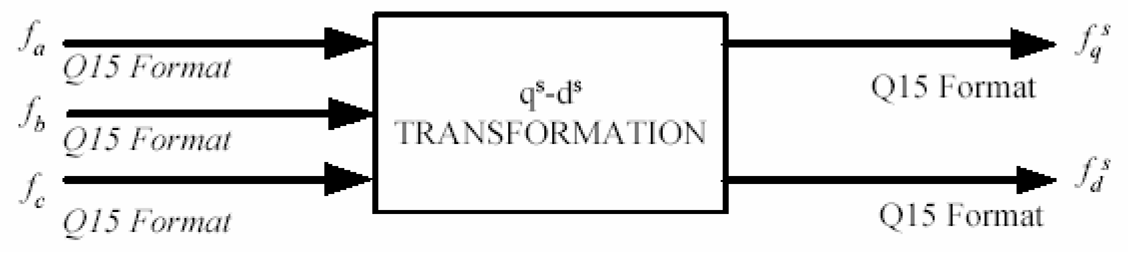
- The first transformation is dual to Clarke's transformation
  - but the q^s axis is in the direction of α -axis, and d^s axis is in negative direction of β -axis.

Implementation of Park's Transformation a-b-c to q^s-d^s

- This transformation transfers the three-phase stationary parameters from an a-b-c system to a two-phase orthogonal stationary reference frame.

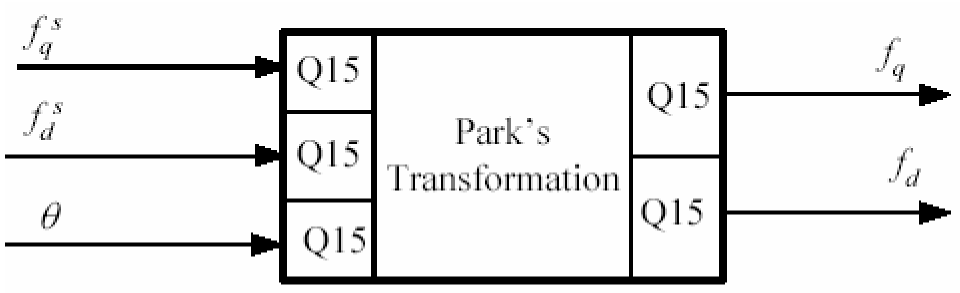
$$f_q^s = f_a$$

$$f_d^s = -\frac{1}{\sqrt{3}}(2f_b + f_a)$$



Implementation of Park's Transformation q^s-d^s to q-d

- This transformation converts vectors in a balanced two-phase orthogonal stationary system into an orthogonal rotary reference frame.



$$f_q = \cos\theta . f_q^s - \sin\theta \cdot f_d^s$$

$$f_d = \sin\theta . f_q^s + \cos\theta \cdot f_d^s$$

Implementation of Park's Transformation q^s-d^s to q-d

- In this transformation, it is necessary to calculate sin θ and cos θ, where the method to calculate them was presented in a previous section.
- All the input and outputs are in the Q15 format and in the range of 8000h-7FFFh

Implementation of Park’s Transformation q^s-d^s to q-d

- The following code is written to implement Park’s transformation:

```
SPM     1             ;SPM set for Q15 multiplication
ZAC                   ;Reset accumulator
LT      f_q_s         ;TREG = f_q_s
MPY     sin_theta     ;PREG = f_q_s * sin(theta)
LTA     f_d           ;ACC = f_q_s * sin(theta) and
                      ;TREG =f_q_s
MPY     cos_theta     ;PREG = f_d_s* cos_teta
MPYA    sin_theta     ;ACC=f_q_s*sin_teta+f_d_s*
                      ;cos_teta andPREG=f_q_s*sin_teta
SACH    park_D        ;f_d =f_q_s * cos_teta + f_d_s*
                      ;sin(theta)
```

Implementation of Park's Transformation q^s-d^s to q-d

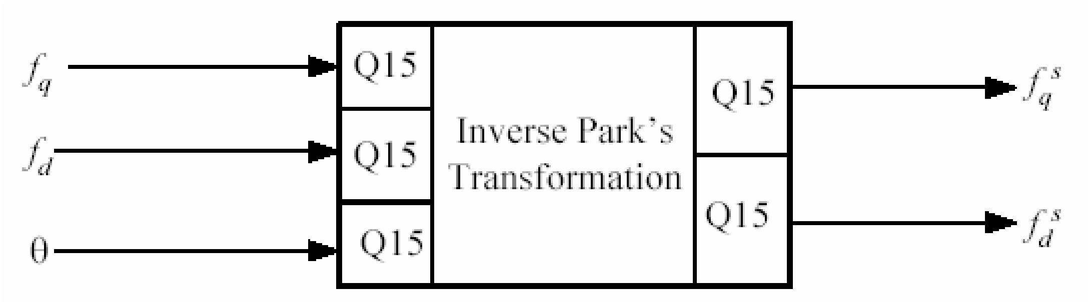
```
LACC    #0            ;Clear ACC
LT      f_d_s         ;TREG = f_d_s
MPYS    cos_theta     ;ACC=- f_d_s* *sin(theta) and
                      ;PREG = f_q_s * cos(theta)
APAC                  ;ACC=- f_d_s*sin(theta) +f_q_s*
                      ;cos(theta)
SACH    f_q           ;fq = -f_d_s*sin(theta) +f_q_s*
                      ;cos(theta)
SPM     0             ;SPM reset
RET
```

Implementation of Park's Transformation q-d to q^s-d^s

- This transformation projects vectors in an orthogonal rotating reference frame into a two-phase orthogonal stationary frame.

$$f_q^s = \cos\theta \cdot f_q + \sin\theta \cdot f_d$$

$$f_d^s = -\sin\theta \cdot f_q + \cos\theta \cdot d_d$$



Implementation of Park's Transformation q-d to q^s-d^s

- The following code is written to implement this transformation:

```
SPM     1           ;SPM set for Q15 multiplication
ZAC                 ;Reset accumulator
LT      f_q         ;TREG = fq
MPY     cos_theta           ;PREG = fq * cos(theta)
LTA     f_d                 ;ACC=fq*cos(theta) and TREG =fd
MPY     sin_theta           ;PREG = fd * sin(theta)
MPYA    sin_theta           ;ACC=fq*cos(theta)+fd*sin(theta) and
                            ;PREG=fd*sin(theta)
SACH    f_q_s               ;fd=fq*cos(theta)+fd*sin(theta)
LACC    #0                  ;Clear ACC
LT      f_d                 ;TREG = fd
MPYS    cos_theta ;ACC = -fd*sin_theta and PREG = fd*cos_theta
APAC
SACH    f_d_s
SPM     0           ;SPM reset
RET
```

Implementation of Park's Transformation q^s-d^s to a-b-c

- This transformation transforms the variables from the stationary two-phase frame to the stationary a-b-c system.

- This system is also dual to the inverse Clarke transformation where the q^s-axis is in the direction of the α-axis and the d^s-axis is in the negative direction of β-axis.

$$f_a = f_q^s$$

$$f_b = \frac{-f_q^s - \sqrt{3} f_d^s}{2}$$

$$f_c = \frac{-f_q^s - \sqrt{3} f_d^s}{2}$$

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Conclusion

- With FOC of synchronous and induction machines, it is desirable to reduce the complexity of the electric machine voltage equations.
- The transformation of machine variables to an orthogonal reference frame is beneficial for this purpose.
- Park's and Clarke's transformations, two revolutions in the field of electrical machines, were studied in depth in this chapter.
- These transformations and their inverses were implemented on the fixed point LF2407 DSP.

Reading materials

- Bpra047 - Sine, Cosine on the TMS320C2xx
- Bpra048 - Clarke & Park Transforms on the TMS320C2xx
- Spru485a - *Digital Motor Control Software Library*
  - CLARKE,
  - PARK,
  - I_CLARKE,
  - I_PARK,